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U.S. Army Intelligence Center and School Software Analysis and Management System

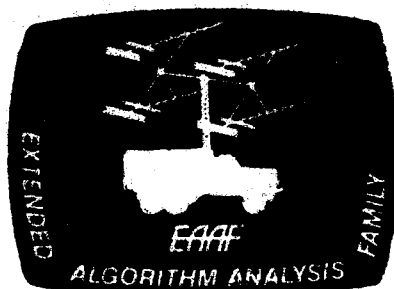## Use of Distance Versus Statistical Distance in Acceptance Tests

Technical Memorandum No. 35

MARC

Mathematical Analysis Research Corporation

EAAF

EXTENDED ALGORITHM ANALYSIS FAMILY

29 September 1987

National Aeronautics and
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JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

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Acceptance Test, Covariance Matrix, Efficiency of Test, Fix Combination, Error Ellipse

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*
The problem treated in this report is that of deciding, with a given (50%) confidence level and based on distance alone, whether two estimated fixes belong to the same emitter. In place of geometric distance – $(x-y)^T(x-y)$ – the authors propose use of statistical distance – $(x-y)^T B^{-1}(x-y)$ – where B is a covariance matrix computed from those matrices of the fixes. The two methods are compared for various ratios of the axes of the 50% probability ellipse.

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U.S. ARMY INTELLIGENCE CENTER AND SCHOOL
Software Analysis and Management System

Use of Distance Versus Statistical
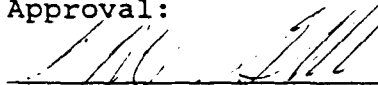Distance in Acceptance Tests

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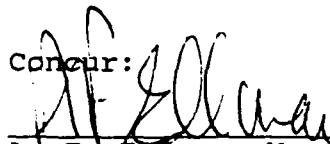
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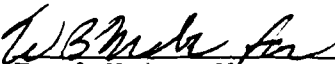
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PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

This specific work was performed in accordance with the FY-87 statement of work (SOW #2).

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Use of Distance Versus Statistical Distance In Acceptance Tests

INTRODUCTION

The proximity of two estimated locations (or one estimate and one known location) is often used as the the basis for judging whether or not to associate the two entities. Using statistical distance is similar except that knowledge of the uncertainty of the locations can be incorporated. In particular, differences in the uncertainty in different directions are taken into account when statistical distance is used. The significance of incorporating the directional information is considered in this report.

DEFINITION OF TECHNIQUES AND ASSUMPTIONS

Let X and Y denote the random variable vectors corresponding to the two estimated locations. Let x and y correspond to the observed vectors for X and Y respectively.

Assume that X and Y correspond to the same true position.

Assume that the estimates X and Y are independently normally distributed distributed with mean equal to the true location.

Thus (X-Y) is normally distributed with mean zero and a covariance matrix, call it B, computable from covariance matrices for X and Y. The details of the calculation are of little interest to this report. For a geometric feeling for B see Figure 1.

Distance Squared$=(x-y)^T(x-y)$

Statistical Distance Squared$=(x-y)^T B^{-1}(x-y)$

There are tests based on these statistics. They involve computing the probability that the distance would be as large as the observed distance assuming they really do belong to one emitter.

DISTANCE TEST:
Compute the probability that $P((X-Y)^T(X-Y)>(x-y)^T(x-y))$.

This probability is computed using the ratio of the eigenvalues of B or its reciprocal whichever is smaller.

STATISTICAL DISTANCE TEST:
Compute the probability that $P((X-Y)^T B^{-1}(X-Y)>(x-y)^T B^{-1}(x-y))$.

OBSERVATIONS

The difference between the DISTANCE TEST and the STATISTICAL DISTANCE TEST depends upon two things:
- i) the direction of (x-y)
- ii) the ratio of the eigenvalues of B

EXTREME CASE #1- Ratio=1.

If the ratio of the eigenvalues is one, then B has no impact and the two metnods are the same.

EXTREME CASE #2- Ratio close to zero and (x-y) is in the direction of the smaller EEP axis.

In this case the DISTANCE TEST is much more likely to accept two fixes as being from the same emitter than the STATISTICAL DISTANCE TEST. See Figure 2.

EXTREME CASE #3- Ratio close to zero and (x-y) is in the directicn of the larger EEP axis.

In this case the DISTANCE TEST is slightly less likely to accept two fixes as being from the same emitter than the STATISTICAL DISTANCE TEST. The amount less likely depends on the actual cut-off level being used. See Figure 2.

In all cases except the ratio=1 case once the Probability that will be accepted is set, the area where the DISTANCE TEST accepts is larger than the area where the STATISTICAL DISTANCE TEST accepts. This results from the fact the DISTANCE TEST orders points by distance whereas the STATISTICAL DISTANCE TEST orders points by likelihood. As an example, if the 50% probability cut off value is used then the following effeciencies result:

EFFICIENCIES USING A 50% PROBABILITY OF CAPTURING THE TRUE AS A CUTOFF

Eigenvalue Ratio	Efficiency	
∞ or 0	0%	
99 or 1/99	29.95%	$= 100[1((1/99)^{.5})(-2\ln(.5))/(.46056(1+1/99))]$
19 or 1/19	61.91%	$= 100[1((1/19)^{.5})(-2\ln(.5))/(.48799(1+1/19))]$
9 or 1/9	78.29%	$= 100[1((1/9)^{.5})(-2\ln(.5))/(.53123(1+1/9))]$
7 or 1/7	83.01%	$= 100[1((1/7)^{.5})(-2\ln(.5))/(.55228(1+1/7))]$
4 or 1/4	91.49%	$= 100[1((1/4)^{.5})(-2\ln(.5))/(.60610(1+1/4))]$
3 or 1/3	94.69%	$= 100[1((1/3)^{.5})(-2\ln(.5))/(.63392(1+1/3))]$
7/3 or 3/7	96.86%	$= 100[1((3/7)^{.5})(-2\ln(.5))/(.65586(1+3/7))]$
2 or 1/2	97.90%	$= 100[1((1/2)^{.5})(-2\ln(.5))/(.66749(1+1/2))]$
1.5 or 2/3	99.28%	$= 100[1((2/3)^{.5})(-2\ln(.5))/(.68403(1+2/3))]$
1	100.00%	$= 100[1((1/1)^{.5})(-2\ln(.5))/(.69315(1+1/1))]$

These tables would have to be recalculated for other acceptance probabilities if other acceptance probabilities were being used.

CONCLUSIONS:

The STATISTICAL DISTANCE is the more powerful test. It is less likely to accept a fix from another source at any given probability cut off. (This probability cut-off is directly related to the probability of failing to accept two fixes from a common source for combination.) The EFFICIENCY of the DISTANCE TEST can be determined as the ratio of the areas of the elliptical region of acceptance of the STATISTICAL DISTANCE TEST to the circular region of acceptance of the DISTANCE TEST.

The ellipses determine the STATISTICAL DISTANCE acceptance region.

The STATISTICAL DISTANCE acceptance region determines the DISTANCE TEST acceptance region.

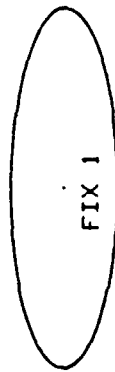
For example in the region below the ratio of the STATISTICAL DISTANCE axes is 3 to 1. Squaring implies the eigenvalue ratio was 9 and hence the STATISTICAL DISTANCE test only uses 78.29% as much area as the DISTANCE test.
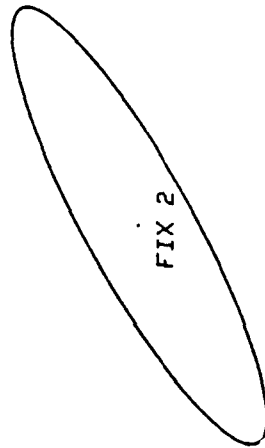

50% Acceptance Region For the DISTANCE test
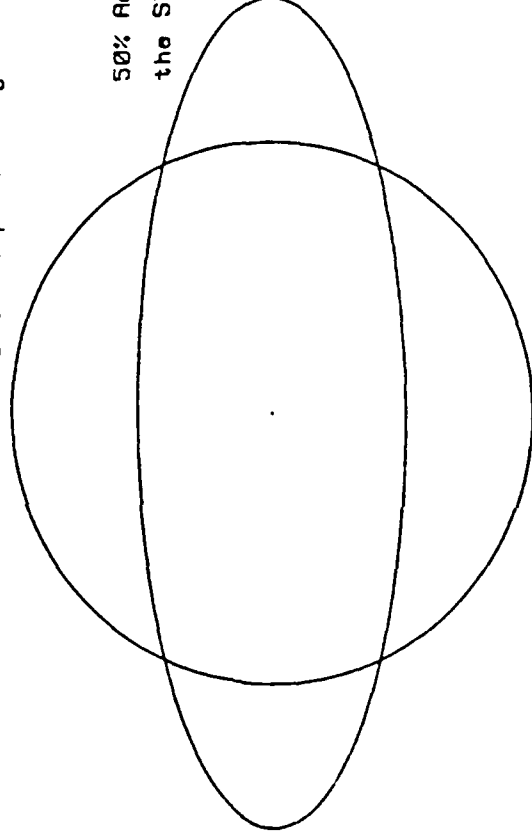
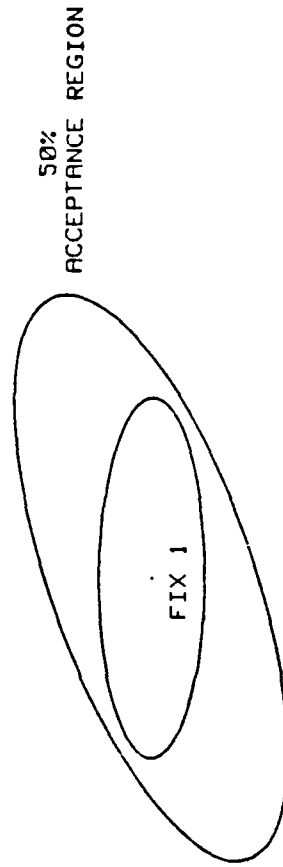50% Acceptance Region for the STATISTICAL DISTANCE test

Given an existing 50% ellipse ...

FIX 1

And given the shape and size of the 50% ellipse of an incoming ellipse

FIX 2

Then there is an acceptance region about the first fix where the incoming fix is accepted by the STATISTICAL DISTANCE TEST

50%
ACCEPTANCE REGION

FIX 1

END

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